

Summer Scholars Project Proposal

Title

Investigations in Combinatorics: Pólya-DeBruijn Theory and Burnside's Lemma

Student

XXXX

Faculty Mentor

Professor XXXX

Department of Mathematics, Saint Joseph's University

Relevant Coursework Already Completed

MAT 1431 Theory of Numbers

MAT 1591 The Fundamental Ideas of Mathematics

MAT 2031/41 Modern Algebra I & II

MAT 2441 Combinatorics and Graph Theory

Background

In how many ways can you paint the faces of a cube, starting with k colors of paint? The answer is difficult, because not all of the k^6 ways of painting the six faces count as different; symmetries of the cube make some colorings equivalent to others. In mathematical terms, there is a group inducing an equivalence relation on the ways of painting the cube, and what is wanted is a way to count the equivalence classes.

The first result along these lines is Burnside's Lemma (actually discovered by Frobenius), which gives a formula for the number of orbits that result when any group acts on any set. Pólya used Burnside's Lemma to find a useful expression for something called the "pattern inventory" (relative to a "weight function") for colorings like the above; this expression includes as a special case the number of colorings. Generalizing Pólya's method, DeBruijn found a way to count the colorings when groups are acting both on the domain of the colorings (in the example, the symmetries of the cube) and the range (the set of colors).

Description of the Proposed Project

I propose to learn the mathematics behind these results and to learn to apply these results to specific counting problems. The main source for the mathematics will be a detailed article by DeBruijn himself. This project is particularly important to me because I plan to work on a Senior Honors project in which I will be employing these techniques.